Asymptotic symmetry of geometries with Schrödinger isometry

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Abstract

We show that the asymptotic symmetry algebra of geometries with Schrödinger isometry in any dimension is an infinite dimensional algebra containing one copy of Virasoro algebra. It is compatible with the fact that the corresponding geometries are dual to non-relativistic CFTs whose symmetry algebra is the Schrödinger algebra which admits an extension to an infinite dimensional symmetry algebra containing a Virasoro subalgebra.

1   Introduction

AdS/CFT correspondence [1] has provided us with a powerful framework to study strongly coupled conformal field theories. This is done by making use of weakly coupled gravities on backgrounds containing an AdS part. According to the AdS/CFT duality there is a one to one correspondence between objects on the gravity side and those in the dual conformal field theory. In particular it is known that the symmetries of the conformal field theory can be geometrically realized on the gravity side as the isometries of the metric. Indeed it is well known that the asymptotic symmetry of an AdS_{d+1} geometry is the conformal group in d dimensional spacetime [2].

For later use, following [2], it is instructive to review how to find the asymptotic symmetry of a geometry which is asymptotically AdS. Consider a metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ where $\bar{g}_{\mu\nu}$ is the metric of an AdS_{d+1} space given by

$$ds^2 = \bar{g}_{mn}dx^m dx^n = \frac{-dt^2 + dx_i^2 + dz^2}{z^2}, \qquad i = 1, \cdots, d-1. \tag{1.1}$$

and $h_{\mu\nu}$ is deviation of the metric consistent with the particular solution. To have a well defined asymptotically AdS solution $h_{\mu\nu}$ should have a proper falloff as we approach the boundary at $z \to 0$. We consider the boundary conditions as follows [2]

$$h_{\mu\nu} = \mathcal{O}(1), \qquad h_{z\mu} = \mathcal{O}(z), \qquad h_{zz} = \mathcal{O}(1), \tag{1.2}$$

where $\mu, \nu = 0, 1, \cdots d-1$. Now the aim is to find the asymptotic Killing vectors $\xi = \xi^m \partial_m$ which preserve the above boundary conditions. It is straightforward to solve the asymptotic Killing equations, $\mathcal{L}_\xi \bar{g}_{mn} = h_{\mu\nu}$, to find [3]

$$\begin{aligned}
\xi^\mu &= \epsilon^\mu(t, x_i) - \frac{z^2}{d}\eta^{\mu\nu}\partial_\nu\left(\sum_\rho \partial_\rho \epsilon^\rho(t, x_i)\right) + \mathcal{O}(z^4), \\
\xi^z &= \frac{z}{d}\sum_\rho \partial_\rho \epsilon^\rho(t, x_i) + \mathcal{O}(z^3),
\end{aligned} \tag{1.3}$$

where $\eta^{\mu\nu}$ is the metric of the flat Minkowski space and ϵ^μ is a function of t and x_i satisfying

$$\partial_\mu \epsilon_\nu(t, x_i) + \partial_\nu \epsilon_\mu(t, x_i) = \frac{2}{d}\sum_\rho \partial_\rho \epsilon_\rho. \tag{1.4}$$

We note that this is the equation we get for the parameter of the conformal transformation in d dimensions. When $d > 2$ this leads to $SO(2, d)$ group while for $d = 2$ the conformal group generates two copies of the Virasoro algebra.

The aim of this article is to extend the above considerations for geometries with Schrödinger isometry. In fact it has recently been proposed in [4, 5] that gravity on a background with Schrödinger isometry is dual to a non-relativistic CFT.

An important point we would like to mention is that the $d-1$ dimensional Schrödinger algebra which generates the symmetry of the d dimensional non-relativistic CFTs admits an extension to an infinite dimensional symmetry algebra containing a Virasoro subalgebra.

This has to be compared with the relativistic CFTs whose symmetry algebra is finite in all dimensions except for $d = 2$, where we get two copies of Virasoro algebra. Therefore if we would like to establish the non-relativistic AdS/CFT correspondence one should be able to show that the asymptotic symmetry of geometries with Schrödinger isometry is an infinite dimensional algebra containing one copy of the Virasoro algebra in arbitrary dimensions. In this article we show how this can happen.

This paper is organized as follows. In the next section we briefly review the $d - 1$ dimensional Schrödinger algebra and its infinite dimensional extension. In section three we study the asymptotic symmetry of the geometries with Schrödinger isometry where we show that an infinite dimensional symmetry can be realized. To be precise in this section we will obtain the algebra of the asymptotic Killing vectors. We note, however, that the conserved charges generating the asymptotic symmetry in general have the same algebra up to central extension. The last section is devoted to discussions.

2 Non-relativistic CFTs and Schrödinger algebra

In this section we review the Schrödinger algebra and its infinite dimensional extension in arbitrary dimensions. The generators of the Schrödinger algebra are spatial translations P_i, rotations J_{ij}, time translation H, Galilean boosts K_i, dilation D and special conformal transformation C. The algebra of Sch_{d-1} with a central extension given by the number operator N may be written as [1]

$$
\begin{aligned}
&[J_{ij}, P_k] = -(\delta_{ik}P_j - \delta_{jk}P_i), &\quad &[J_{ij}, K_k] = -(\delta_{ik}K_j - \delta_{jk}K_i), \\
&[J_{ij}, J_{kl}] = -\delta_{ik}J_{jl} + \text{perms}, &\quad &[P_i, K_j] = -N\delta_{ij} \\
&[D, P_i] = \frac{1}{2}P_i, \quad [D, K_i] = -K_i, \quad [D, H] = H, \\
&[C, P_i] = K_i, \quad\quad [D, C] = -C, \quad [C, H] = 2D.
\end{aligned} \tag{2.1}
$$

It is worth noting that the generators D, H and C form an $SL(2, R)$ algebra. As we will see this is exactly the reason why the Schrödinger algebra has an infinite dimensional extension. On the other hand since $[D, N] = 0$ one may diagonalize them simultaneously leading to the fact that representations of the Schrödinger algebra may be labeled by two numbers; conformal dimension Δ and a number M which are the eigenvalues of D and N, respectively.

The generators of the Schrödinger algebra can be thought of as vector fields defined on d dimensional spacetime with the following representation

$$
\begin{aligned}
J_{ij} &= -(x_i\partial_j - x_i\partial_i), \quad P_i = -\partial_i, \quad H = -\partial_t, \quad N = -M \\
K_i &= -(t\partial_i + x_iM), \quad D = -(t\partial_t + \frac{1}{2}x_i\partial_i), \quad C = -(t^2\partial_t + tx_i\partial_i + \frac{1}{2}x^2M). \tag{2.2}
\end{aligned}
$$

where $x^2 = x_ix_i$. Following [10] one can define the generators of the corresponding infinite dimensional algebra in $d - 1$ dimensions as follows

$$
L_n = -t^{n+1}\partial_t - \frac{n+1}{2}t^nx_i\partial_i - \frac{n(n+1)}{4}t^{n-1}x^2M,
$$

---

[1]This algebra can be obtained from the relativistic conformal algebra in $d + 1$ dimensions. In other words the Schrödinger group may be thought of as a subgroup of $SO(2, d+1)$ with fixed momentum along the null direction (see for example [6–9]).

$$Q_{i\hat{n}} = -t^{\hat{n}+1/2}\partial_i - (\hat{n}+\frac{1}{2})t^{\hat{n}-1/2}x_i M, \quad T_n = -t^n M. \tag{2.3}$$

Here $n \in \mathbb{Z}$ and $\hat{n} \in \mathbb{Z}+\frac{1}{2}$. It is straightforward to see that the above generators satisfy the following commutation relations

$$[L_n, L_m] = (n-m)L_{n+m}, \qquad [Q_{i\hat{n}}, Q_{j\hat{m}}] = (\hat{n}-\hat{m})\delta_{ij}T_{\hat{n}+\hat{m}},$$
$$[L_n, Q_{i\hat{m}}] = (\frac{n}{2}-\hat{m})Q_{i(n+\hat{m})}, \qquad [L_n, T_m] = -mT_{n+m}. \tag{2.4}$$

Note that due to non-trivial contribution of number operator to the Galilean boost the Schrödinger algebra does not allow an infinite dimensional extension for rotations J_{ij}. Thus we find

$$[J_{ij}, Q_{k\hat{n}}] = -(\delta_{ik}Q_{j\hat{n}} - \delta_{jk}Q_{i\hat{n}}), \qquad [L_n, J_{ij}] = 0. \tag{2.5}$$

This may be compared with the Galilean conformal algebra recently studied in [11] where it was shown that the algebra allows an infinite dimensional extension for rotations too.

In the next section we show that the above algebra is indeed asymptotic symmetry algebra of the geometries with Schrödinger isometry.

3 Asymptotic symmetry

Following the relativistic AdS/CFT correspondence one expects that if there exist gravity duals to non-relativistic CFTs, the isometry of the relevant geometry must be the Schrödinger group. In fact such gravity duals exist, for example consider the metric

$$ds^2 = -\frac{dt^2}{z^4} + \frac{-dtd\eta + dx_i^2 + dz^2}{z^2}, \quad i = 1, \cdots, d-1, \tag{3.1}$$

which could be thought of as a solution of a $d+2$ dimensional gravity coupled to massive gauge field [4,5]. It is easy to see that the isometry of the above metric has $d-1$ dimensional Schrödinger group.

This metric has been proposed to provide a gravity description for non-relativistic CFTs. Such non-relativistic CFTs admit a Schrödinger algebra. On the other hand since the Schrödinger algebra admits an infinite dimensional extension one may wonder if the extended algebra can be seen from the gravity description too. The situation is very similar to the AdS_3 case where it is known that its asymptotic symmetry algebra contains two copies of Virasoro algebra in agreement with the symmetry of 2D CFT.

In this section we study the asymptotic symmetry of the metric (3.1) to see how the infinite dimensional symmetry algebra arises. It has to be compared with the AdS space where the symmetry is finite dimensional except for AdS_3 and AdS_2.

The non-trivial point which allows the non-relativistic CFTs to have infinite dimensional symmetry algebra is the fact that the Schrödinger algebra in any dimension has an $SL(2,R)$ subalgebra generated by the time translation, dilation and special conformal transformation. This factor can be extended to the Virasoro algebra and therefore to get a closed algebra the other generators should also admit an infinite dimensional extension.

From the gravity point of view there is a direct observation to see why the metric (3.1) leads to an infinite dimensional symmetry in any dimension while it is not the case for AdS

geometry. In fact the crucial point is to see how the $SL(2,R)$ factor may be realized from the geometry (3.1). To see this, it is useful to define a new coordinate $\rho = z^2/2$ by which the metric (3.1) reads

$$ds^2 = \frac{1}{4}\left(\frac{-dt^2 + d\rho^2}{\rho^2}\right) + \frac{1}{2\rho}(-dtd\eta + dx_i^2). \tag{3.2}$$

It is now evident that for small ρ the first factor dominates and therefore we observe that asymptotically the background approaches an AdS_2 which is the realization of the $SL(2,R)$ subalgebra. On the other hand it has been shown [12] that the asymptotic symmetry group of an asymptotically AdS_2 geometry is one copy of the Virasoro algebra. This is indeed the Virasoro subgroup we are looking for[2].

We note, however, that in our case having had an AdS_2 factor, though necessary, is not sufficient to get infinite dimensional extension of the isometry of the solution. This is due to the fact that the geometry is not asymptotically factorized as a direct product of $AdS_2 \times M_d$ with M_d being d-dimensional Minkowski space. In particular we note that for arbitrary dynamical scaling it is always possible to bring the metric into a form with a factor of AdS_2, though only for the present case we have infinite dimensional extension of the isometry algebra[3]. Therefore to find the explicit form of the generators of the asymptotic symmetry we follow the procedure of [2] as reviewed for AdS space in the introduction.

To proceed, motivated by the boundary conditions of the AdS space (1.2), we consider the following boundary conditions for geometries which are asymptotically equivalent to the metric (3.1)

$$\begin{pmatrix} h_{tt} = \mathcal{O}(\frac{1}{z^2}) & h_{t\eta} = \mathcal{O}(1) & h_{ti} = \mathcal{O}(z^2) & h_{tz} = \mathcal{O}(z) \\ h_{\eta t} = h_{t\eta} & h_{\eta\eta} = \mathcal{O}(z^4) & h_{\eta i} = \mathcal{O}(z^2) & h_{\eta z} = \mathcal{O}(z) \\ h_{it} = h_{ti} & h_{i\eta} = h_{\eta i} & h_{ij} = \mathcal{O}(1) & h_{iz} = \mathcal{O}(z) \\ h_{zt} = h_{tz} & h_{z\eta} = h_{\eta z} & h_{zi} = h_{iz} & h_{zz} = \mathcal{O}(1) \end{pmatrix}. \tag{3.3}$$

It is straightforward to see that the following asymptotic Killing vectors preserve the above boundary condition

$$\begin{aligned} \xi^{(1)} &= \left[\epsilon(t) + \mathcal{O}(z^6)\right]\partial_t + \left[\frac{z}{2}\epsilon'(t) + \mathcal{O}(z^3)\right]\partial_z + \left[\frac{x_i}{2}\epsilon'(t) + \mathcal{O}_i(z^4)\right]\partial_i \\ &+ \left[\delta(t) + \frac{z^2 + x^2}{4}\epsilon''(t) + \mathcal{O}(z^4)\right]\partial_\eta, \\ \xi_i^{(2)} &= \mathcal{O}_i(z^6)\partial_t + \mathcal{O}_i(z^3)\partial_z + \left[\beta(t) + \mathcal{O}(z^4)\right]\partial_i + \left[\beta'(t)x_i + \mathcal{O}_i(z^4)\right]\partial_\eta, \end{aligned} \tag{3.4}$$

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[2]Galilean conformal algebra in d dimensions has recently been obtained from d dimensional conformal algebra by a contraction in [11]. The authors observed that the Galilean conformal algebra admits an extension to an infinite dimensional symmetry algebra. In the light of our discussions one may wish to interpret this effect as the fact that there is an $SL(2,R)$ subgroup which should also be geometrically realized from the gravity description. Actually this is the case. Indeed the corresponding gravity description may also be obtained by a contraction of AdS_{d+1} geometry. Doing this the resultant gravity develops an AdS_2 factor as the base space over which we have a $d-1$ dimensional fiber.

[3]We would like to thank M. Edalati for pointing out the confusion of the text in the early version of the paper.

where ϵ, β and δ are functions of t which may be thought of as independent polynomials of t. The prime denotes derivative with respect to t. To mode expand the above Killing vectors we set $\epsilon(t) = -t^{n+1}, \beta(t) = -t^n$ and $\delta(t) = -t^n$ by which one finds

$$
\begin{aligned}
L_n &= -t^{n+1}\partial_t - \frac{n+1}{2}t^n(z\partial_z + x_i\partial_i) - \frac{n(n+1)}{4}t^{n-1}(z^2 + x^2)\partial_\eta, \\
Q_{in} &= -(t^n\partial_i + nt^{n-1}x_i\partial_\eta), \qquad T_n = -t^n\partial_\eta,
\end{aligned}
\tag{3.5}
$$

satisfying

$$
\begin{aligned}
[L_n, L_m] &= (n - m)L_{n+m}, & [Q_{i\hat{n}}, Q_{j\hat{m}}] &= (\hat{n} - \hat{m})\delta_{ij}T_{\hat{n}+\hat{m}}, \\
[L_n, Q_{i\hat{m}}] &= (\frac{n}{2} - \hat{m})Q_{i(n+\hat{m})}, & [L_n, T_m] &= -mT_{n+m},
\end{aligned}
\tag{3.6}
$$

in precise agreement with (2.4). In order to compare our results with those in the previous section we have denoted the generators of the asymptotic Killing vectors by the same latter as that in (2.3). Note that to precisely map the resultant algebra to those in (2.4) one needs to change $n \to n + \frac{1}{2} \equiv \hat{n}$ in the generators Q_{in}.

It is worth mentioning that for the limit of $z \to 0$, setting $\partial_\eta = M$ the above generators reduce to those in the previous section. This is compatible with the fact that the derivative ∂_η should be identified with the number operator in the dual non-relativistic CFT (see for example [13–15]). It is also obvious from the metric (3.1) that it is invariant under rotations among the spacial coordinates x_i's giving rise to the rotation generators J_{ij} as the last part of the symmetry algebra.

As a result we could uncover the infinite dimensional Schrödinger algebra from the algebra of asymptotic Killing vectors of metric (3.1). On the other hand we note that the conserved charges of the corresponding asymptotic Killing vectors which generate the asymptotic symmetry satisfy the same algebra of the asymptotic Killing vectors up to a possible central extension. In the next section we will give comments on the central extension of the resultant algebra.

4 Discussions

In this paper we have studied the asymptotic symmetry of geometries with Schrödinger isometry. We have shown that unlike the AdS geometry where only for AdS_3 and AdS_2 we get infinite dimensional asymptotic symmetry, in this case the asymptotic symmetry is infinite dimensional in any dimension.

We have noticed that the crucial point behind this interesting phenomenon is the fact that the Schrödinger group in any dimension has an $SL(2, R)$ subgroup which can be geometrically realized from the dual gravity description. More precisely geometries with Schrördinger isometry are asymptotically AdS_2. It is also known that the asymptotic symmetry of the AdS_2 space is one copy of Virasoro algebra. As a result the $SL(2, R)$ subalgebra can be extended to the Virasoro algebra. In fact to find the infinite dimensional extension of the algebra it is enough to find the Virasoro algebra. The others can be found via the closure of the algebra.

As a byproduct, our results make it possible to compare two different proposals of the gravity description of non-relativistic CFTs made in [4, 5] and [16, 17]. While the former

proposal is based on the metric (3.1), the latter is based on gravity in the AdS geometry where the conformal symmetry of the boundary is broken to the non-relativistic one via a non-trivial boundary condition for the fields on the bulk. As we reviewed in the introduction the asymptotic symmetry of the AdS space is a finite dimensional algebra which does not allow to get an infinite dimensional extension of the Schrödinger algebra. Therefore our result is in favor of the proposal based on the metric (3.1) [4,5].

Since we have a Virasoro subalgebra an immediate question arises, does this Virasoro algebra admit a central extension? In other words we would like to know if the algebra of conserved charges which generates the above asymptotic symmetry has non-zero central charge. Actually it is known that there is a unique central extension to the infinite dimensional Schrödinger algebra [10] and indeed the central extension is the one parameterized by central charge c of the Virasoro subgroup, *i.e.*

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m}. \tag{4.1}$$

On the other hand since this factor is the common part of the Schrödinger algebra in any dimension, one may expect that the non-relativistic CFTs have non-zero central charge in any dimension. Actually it was shown [18] that the non-relativistic field theories may give rise to anomalies.

From the gravity point of view since in any dimension the geometry develops an AdS_2 factor as we approach the boundary, following [19] (see also [20–24]) one may expect that the gravity on asymptotic AdS_2 geometry leads to a Virasoro algebra with non-zero central charge. Of course the argument of [19] was based on the fact that there is a twisted energy momentum tensor due to a non-zero $U(1)$ current. Since the metric we have been considering may be embedded in a $d + 2$ dimensional gravity coupled to a massive gauge filed [4], one may suspect that the same argument as that in [19] may be applied in our case too leading to a non-zero central charge. It would be interesting to understand this point better.

To have an insight how the Virasoro algebra may get a central extension it would be instructive to study $d = 1$ in more detail where we will also observe a new phenomenon. For $d = 1$ the metric (3.1) reads

$$ds^2 = -\frac{dt^2}{z^4} + \frac{-dtd\eta + dz^2}{z^2}, \tag{4.2}$$

with the following asymptotic symmetry algebra

$$[L_n, L_m] = (n - m)L_{n+m}, \qquad [L_n, T_m] = -mT_{n+m}. \tag{4.3}$$

The above solution has $SL(2, R) \times U(1)$ isometry which is naively extended to a Virasoro algebra plus an affine $U(1)$ algebra.

To find the central charge it is useful to note that the above solution may be alternatively thought of as a solution of topologically massive gravity known as null AdS_3 solution [25]. It was argued in [25] that the CFT dual to this background at least has one copy of Virasoro algebra with central charge $2/G$, where G is three dimensional Newton's constant[4].

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[4]In our notation we set the radius of the metric to one.

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In this context it has been shown [26] that the symmetry of the conserved charges may contain two copies of Virasoro algebra with the following central charges[5]

$$c_+ = \frac{1}{G}, \qquad c_- = \frac{2}{G},$$
(4.4)

Therefore, if correct, it means that in three dimensions we get even more than what we bargained for; namely not only the central charge is non-zero but the geometry leads to two Virasoro algebras with non-zero central charges.

This has to be compared with higher dimensional cases where it was proved that the Schrödinger algebra can have only one central term corresponding to the central extension of *one* copy of Virasoro algebra which appeared in the infinite dimensional extension of the Schrödinger algebra [10]. The point which might be responsible for this peculiar behavior is that for $d = 1$ due to the absence of the spacial directions it is possible to impose another falloff of the metric components such that to give way for bigger symmetries [26] which is impossible when Q_i's are non-zero.

Concerning the above discussions one may conclude that similar to the relativistic CFTs for which $d = 2$ is exceptional in the sense that conformal group becomes infinite dimensional in two dimensions, the non-relativistic CFT's in $d = 1$ are also exceptional. While the non-relativistic conformal symmetry in any dimension has an infinite dimensional extension, it is only the case of $d = 1$ where we get two copies of Virasoro algebra. It would be extremely interesting to further explore this point.

Acknowledgments

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